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Muon decays with lepton–number violation via vector leptoquark

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Abstract

The decays $\mu \to e\gamma$, $\mu \to e\gamma\gamma$, and $\mu \to ee\bar{e}$ are analysed in the framework of the Pati-Salam type quark-lepton symmetry $SU(4)_V \otimes SU(2)_L \otimes G_R$ where the effects of mixing in the quark-lepton currents are taken into account. It is shown that the $\mu \to e\gamma\gamma$ and $\mu \to ee\bar{e}$ decays via the vector leptoquark have not a GIM–type suppression, while the $\mu \to e\gamma$ decay has. So, the specific hierarchy of the decay probabilities could take place $\Gamma(\mu \to ee\bar{e}) \gg \Gamma(\mu \to e\gamma\gamma) \gg \Gamma(\mu \to e\gamma)$. The existing bounds on the vector leptoquark mass and on the mixing matrix elements, based on the data for the μe conversion in nuclei and for the ratio of the K_{e2} and $K_{\mu 2}$ decays allow to set the upper limits on the branching ratios at a level of 10^{-18} for the $\mu \to e\gamma\gamma$ decay and at a level of 10^{-15} for the $\mu \to ee\bar{e}$ decay.

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All existing experimental data in particle physics are in good agreement with the Standard Model predictions. However, the problems exist which could not be resolved within the SM comprehensively. A phenomenon of the fermion mixing in the charged weak currents appears to be one of the most intriguing of them. An effect of the mixing in the quark sector is depicted by the Cabibbo–Kobayashi–Maskawa unitary (3×3) matrix V_{ij}. It is measured with rather high accuracy [1], and the information on mixing parameters is being permanently improved. It would be natural to expect of the analogous mixing phenomenon to take place in the lepton sector, provided the neutrino mass spectrum is not degenerated. The neutrino oscillation experiments [2] are the main source of information on the lepton mixing parameters. The lepton–number violating decays, such as $\mu \to e\gamma$, $\mu \to e\gamma\gamma$, $\mu \to ee\bar{e}$ [3] are also under the intensive experimental searches. Let us point out, however, that these decay modes are strongly suppressed in the SM due to the well–known GIM cancellation by the factor

$$\left(\frac{m_\nu}{m_W}\right)^4 \sim 10^{-39} \cdot \left(\frac{m_\nu}{20\,eV}\right)^4 , \tag{1}$$

see, e.g. Refs. [4, 5]. Due to the smallness of neutrino mass, a conclusion is forced for the processes having this width to be unobservable in a laboratory. If an existence of the fourth generation is assumed, the neutral lepton of it must be heavy, $m_{L^0} > m_Z/2$, and the suppression of the type of Eq. (1) disappears. It should be noted that another kind of suppression could arise in this case by the small mixing angles. Really, a noticeable mixing of the leptons of the 4th generation with the 1st and the 2nd has to violate the $\mu - \beta$ universality and to cause an effect of the non-orthogonality of the phenomenological electron and muon neutrinos [6].

There exists an alternative possibility of the Standard Model extension where the above–mentioned rare muon decays could arise without the GIM suppression. It is the Minimal Quark–Lepton Symmetry of the Pati–Salam type based on the gauge group $SU(4)_V \otimes SU(2)_L \otimes G_R$ with lepton number as the fourth color [7]. In the recent parers [8] the possible low–energy manifestations of this symmetry were analysed. There exist the most exotic objects as the fractionally–charged and colored gauge X bosons named leptoquarks which cause the interconversions of quarks and leptons. As was shown in [8], a new type of mixing in the quark-lepton current interactions with leptoquarks has to be taken into consideration. An additional arbitrariness of

the mixing parameters could allow, in principle, to decrease noticeably the lower bound on the vector leptoquark mass M_X originated from the rare π and K decays [1]. The only mixing independent bound emerging from the cosmological limit on the $\pi^0 \to \nu\bar{\nu}$ decay width [9] is $M_X > 18\ TeV$.

Let us investigate the contribution of the leptoquark interactions to the decays $\mu \to e\gamma$, $\mu \to e\gamma\gamma$ and $\mu \to ee\bar{e}$. The corresponding part of the Lagrangian of the *down*–type fermion interaction with the leptoquark has the form

$$\mathcal{L}_X = \frac{g_S(M_X)}{\sqrt{2}} [\mathcal{D}_{\ell n} (\bar{\ell}\gamma_\alpha d_n^c) X_\alpha^c + h.c.] \,, \tag{2}$$

where c is the $SU(3)$ color index, the indices ℓ and n correspond to the *down*–fermions, namely, charged leptons $\ell = e, \mu, \tau$ and quarks $n = d, s, b$. The constant $g_S(M_X)$ can be expressed in terms of the strong coupling constant α_S at the leptoquark mass scale M_X, $g_S^2(M_X)/4\pi = \alpha_S(M_X)$.

If the momentum transferred is $q \ll M_X$, then the Lagrangian (2) leads to an effective four-fermion interaction of the quark–lepton vector currents. By using the Fiertz transformation, lepton and quark currents of the scalar, pseudoscalar, vector and axial-vector types may be separated in the effective Lagrangian. Let us note that the construction of the effective lepton-quark interaction Lagrangian requires taking account of the QCD corrections estimated by known techniques [10, 11]. In our case the leading log approximation $ln(M_X/\mu_0) \gg 1$ with $\mu_0 \sim 1\ GeV$ to be the typical hadronic scale is quite applicable. Then the QCD correction amounts to the appearance of the magnifying factor $Q(\mu_0)$ at the scalar and pseudoscalar terms

$$Q(\mu_0) = \left(\frac{\alpha_S(\mu_0)}{\alpha_S(M_X)} \right)^{4/\bar{b}} . \tag{3}$$

Here $\alpha_S(\mu_0)$ is the effective strong coupling constant at the hadron mass μ_0 scale, $\bar{b} = 11 - \frac{2}{3}\bar{n}_f$, $\bar{n}_f$ is the averaged number of the quark flavors at the scales $\mu_0^2 \le q^2 \le M_X^2$. If the condition $M_X^2 \gg m_t^2$ is valid, then we have $\bar{n}_f \simeq 6$, and $\bar{b} \simeq 7$.

A part of the effective Lagrangian we are interested in, providing the lepton-number nonconserving transitions, has the form

$$
\mathcal{L}_{eff} \;=\; -\frac{2\pi\alpha_S(M_X)}{M_X^2}\,\mathcal{D}_{\ell n}\mathcal{D}^*_{\ell' n'}\,\{\frac{1}{2}\,(\bar{\ell}\gamma_\alpha\gamma_5\ell')(\bar{d}_{n'}\gamma_\alpha\gamma_5 d_n)
$$

$$
+\,(\gamma_5\to 1)\;+\;Q(\mu_0)\,[(\bar{\ell}\gamma_5\ell')(\bar{d}_{n'}\gamma_5 d_n)\;-\;(\gamma_5\to 1)]\}. \tag{4}
$$

Each of the processes under consideration is described by a number of the one-loop Feynman diagrams with virtual d, s, b quarks. The diagrams giving the main contributions to the amplitudes are shown in Figs.1,2,3. It is worthwhile to divide the range of integration over the virtual momentum k in the loops of Figs.1,2,3 into two parts, taking for the dividing point some scale Λ_0 with the perturbative QCD be applicable above it. It seems reasonable to take $\Lambda_0 \sim (2 \div 3)\Lambda_{QCD}$ if we intend to make the estimations in order of magnitude only. Then the decay amplitudes could be represented in the form $\mathcal{M} = \Delta\mathcal{M}^{LD} + \Delta\mathcal{M}^{SD}$. Here $\Delta\mathcal{M}^{SD}$ is the short-distance contribution corresponding to the range of a big virtual momenta $k > \Lambda_0$ where the free-quark approximation is quite applicable. On the other hand, in an estimation of the long-distance contribution $\Delta\mathcal{M}^{LD}$ $(k < \Lambda_0)$ where the perturbative QCD does not work, we use the pole–dominance model.

As the analyses of the radiative muon decays show, the two–photon decay dominates the one–photon decay in the model considered. Really, as the squared momentum transferred is $q^2 \simeq m_\mu^2$, the main contribution to the $\mu \to e\gamma\gamma$ decay amplitude $\Delta\mathcal{M}^{LD}_{2\gamma}$ is obviously originated from the virtual π^0 meson, see Fig.4, being rather close to the mass–shell. It is sufficient in this case to consider the pseudoscalar term only in the effective Lagrangian (4). In these approximations the long–distance contribution to the $\mu \to e\gamma\gamma$ decay amplitude is

$$
\Delta\mathcal{M}^{LD}_{2\gamma} \;\simeq\; -\,\frac{i\alpha\,\alpha_S(M_X)}{2\,M_X^2}\,\mathcal{D}_{ed}\mathcal{D}^*_{\mu d}\,\frac{Q(\mu_0)}{m_d(\mu_0)}\,\frac{1}{1-q^2/m_\pi^2}
$$

$$
\times(\bar{e}\gamma_5\mu)\;f_{1\rho\sigma}\,\tilde{f}_{2\sigma\rho}, \tag{5}
$$

where α is the fine structure constant, $f_{\rho\sigma} = k_\rho\epsilon_\sigma - k_\sigma\epsilon_\rho$ is the Fourier transform of the photon field tensor, $\tilde{f}_{\sigma\rho} = \frac{1}{2}e_{\sigma\rho\alpha\beta}\,f_{\alpha\beta}$ is the dual tensor, $q = k_1 + k_2$ is the total momentum 4–vector of the photon pair, $m_d(\mu_0)$ is the running mass of the d quark at the μ_0 scale. Let us note that the ratio $Q(\mu)/m(\mu)$ is

the renormalization group invariant, since the $Q(\mu)$ function (3) determines also the law of the quark mass running. To the $\mu_0 \simeq 1\ GeV$ scale there correspond the well-known quark current masses $m_u \simeq 4\ MeV, m_d \simeq 7\ MeV$ and $m_s \simeq 150\ MeV$, see e.g. Refs. [12, 13].

The pole approximation gives zero result for the analogous contribution $\Delta\mathcal{M}_{1\gamma}^{LD}$ to the $\mu \to e\gamma$ decay amplitude, because no intermediate meson state exists to pass into a real photon.

It is interesting to note that the short–distance contribution $\Delta\mathcal{M}_{1\gamma}^{SD}$ to the $\mu \to e\gamma$ decay is also strongly suppressed as compared with the corresponding contribution to the $\mu \to e\gamma\gamma$ decay. It can be easily understood from the following qualitative treatment. If the $\mu \to e\gamma$ process was considered in the local limit of the quark–lepton interaction, see Fig.1, then the quark loop would have only one external momentum, namely, the photon momentum q, and the gauge invariant amplitude with the real photon could not be constructed. Additional momenta of the external particles could appear if the non–local effects in the quark–lepton interaction were only considered. However, an extra factor of suppression $\sim (m_b/M_X)^2$ inevitably arises in the amplitude in this case analogously to the well–known GIM suppression, and in order of magnitude we have

$$\frac{\Gamma(\mu \to e\gamma\gamma)}{\Gamma(\mu \to e\gamma)} \sim \frac{\alpha}{\pi} \left(\frac{M_X}{m_b}\right)^4 \gg 1. \tag{6}$$

The exact calculation does confirm this qualitative analysis.

When the short–distance contribution to the $\mu \to e\gamma\gamma$ decay amplitude is calculated, the vector part of the effective Lagrangian (4) does not work due to the Furry theorem. As the analysis shows, the scalar and pseudoscalar parts dominate and give the equal contributions. We obtain

$$\begin{aligned}\Delta\mathcal{M}_{2\gamma}^{SD} \simeq \frac{\alpha\,\alpha_S(M_X)}{3\,M_X^2}\,\mathcal{D}_{eb}\mathcal{D}_{\mu b}^*\,\frac{Q(\mu_0)}{\Lambda_0}\,[(\bar{e}\mu)\ f_{1\rho\sigma}\ f_{2\sigma\rho} \\ -\ i(\bar{e}\gamma_5\mu)\ f_{1\rho\sigma}\ \tilde{f}_{2\sigma\rho}].\end{aligned} \tag{7}$$

In general, a relative sign of the amplitudes (5) and (7) couldn't be established in the approach used. When the estimation in order of magnitude is performed, the interference term of Eq. (5) and of the pseudoscalar part of

Eq. (7) can be omitted for simplicity and the separate contributions of the long and short distances to the $\mu \to e\gamma\gamma$ decay branch can be found

$$Br(\mu \to e\gamma\gamma)^{LD} \simeq \frac{3\,\alpha^2\,\alpha_S^2(M_X)}{16} \left(\frac{m_\mu\,Q(\mu_0)}{m_d(\mu_0)}\right)^2 \times F\left(\frac{m_\mu^2}{m_\pi^2}\right)\left(\frac{|\mathcal{D}_{ed}\mathcal{D}^*_{\mu d}|}{G_F\,M_X^2}\right)^2, \tag{8}$$

$$F(a) = \frac{1}{a^2}\left(\frac{1}{3} - \frac{4}{a} + \frac{4}{a^2}\right) + \frac{2}{a^3}\left(1 - \frac{3}{a} + \frac{2}{a^2}\right)\ln(1-a), \tag{9}$$

$$Br(\mu \to e\gamma\gamma)^{SD} \simeq \frac{\alpha^2\,\alpha_S^2(M_X)}{180}\left(\frac{m_\mu\,Q(\mu_0)}{\Lambda_0}\right)^2\left(\frac{|\mathcal{D}_{eb}\mathcal{D}^*_{\mu b}|}{G_F\,M_X^2}\right)^2, \tag{10}$$

where G_F is the Fermi coupling constant. The magnitude (8) could be estimated from the experimental data on the $\mu - e$ conversion in nuclei [14] being also possible due to the leptoquark exchange. The bound obtained in Ref. [8] is

$$\frac{M_X}{|\mathcal{D}_{ed}\mathcal{D}^*_{\mu d}|^{1/2}} > 670\ TeV. \tag{11}$$

Considering a rather slow increase of the running coupling constant α_S with energy and assuming $\alpha_S(M_X) \sim \alpha_S(100\ TeV) = 0.063$ we obtain the following numerical estimation for the Br^{LD}

$$Br(\mu \to e\gamma\gamma)^{LD} < 1.4 \cdot 10^{-19}. \tag{12}$$

Given the bound (11) an upper limit on the combination of the model parameters $|\mathcal{D}_{eb}\mathcal{D}^*_{\mu b}|/M_X^2$ involved into (10) could be obtained assuming that the τ lepton is associated mainly with the d quark. As was shown in Ref. [15], the experimental data on the ratio $R_{e/\mu} = \Gamma(K^+ \to e^+\nu)/\Gamma(K^+ \to \mu^+\nu)$ [1] gives the most stringent constraint on the model parameters in this case. The calculation of the leptoquark contribution to it is such as for the ratio of the $\pi_{\ell 2}$ decay probabilities [8]. As a result the bound is $M_X/|\mathcal{D}_{es}| > 55\,TeV$.

Taking into account the unitarity of the $\mathcal{D}$ matrix and the other limits on its elements, see Ref. [8], one obtains

$$\frac{M_X}{|\mathcal{D}_{eb}\mathcal{D}^*_{\mu b}|^{1/2}} \, > \, 55 \; TeV. \tag{13}$$

Finally we get

$$Br(\mu \to e\gamma\gamma)^{SD} \; < \; 1.0 \cdot 10^{-18}. \tag{14}$$

A similar analysis of the $\mu \to ee\bar{e}$ decay shows that the short–distance contribution also dominates there as in the $\mu \to e\gamma\gamma$ decay. An amplitude of the process, see Fig.3, could be represented in the form

$$\mathcal{M}_{3e} \, \simeq \, \Delta\mathcal{M}^{SD}_{3e} \, \simeq \, -\,\frac{2\alpha\,\alpha_S(M_X)}{3\,M_X^2}\,\mathcal{D}_{eb}\mathcal{D}^*_{\mu b}\;\ln\frac{m_b}{\Lambda_0}$$

$$\times[(\bar{e}_1\gamma_\alpha\mu)\,(\bar{e}_2\gamma_\alpha e_3) \, - \, (1 \leftrightarrow 2)]. \tag{15}$$

The $\mu \to ee\bar{e}$ decay branch is

$$Br(\mu \to ee\bar{e}) \simeq \frac{2\alpha^2\,\alpha_S^2(M_X)}{3}\,\left(\ln\frac{m_b}{\Lambda_0}\right)^2\,\left(\frac{|\mathcal{D}_{eb}\mathcal{D}^*_{\mu b}|}{G_F\,M_X^2}\right)^2. \tag{16}$$

Within the above restrictions on the model parameters we obtain

$$Br(\mu \to ee\bar{e}) \; < \; 1.0 \cdot 10^{-15}. \tag{17}$$

The $\mu \to ee\bar{e}$ decay via the vector leptoquark was also considered in Ref. [16] within the model independent approach, but the diagram giving the main contribution to the amplitude, see Fig.3, was not analysed there.

In summary, the minimal quark–lepton symmetry $SU_V(4)\otimes SU_L(2)\otimes G_R$ with taking account of the mixing in the quark–lepton currents leads to some interesting predictions about the rare muon decays with a lepton number nonconservation:

i) a peculiar hierarchy of the decay probabilities could take place

$$\Gamma(\mu \to ee\bar{e}) \gg \Gamma(\mu \to e\gamma\gamma) \gg \Gamma(\mu \to e\gamma), \tag{18}$$

see the estimations (6), (12), (14), (17);

ii) the branches of the considered decays do not depend on the neutrino masses. That is to say that these decays are possible even though the neutrino mass spectrum is degenerated, e.g. all the neutrinos are massless.

Although the predicted values of the branches of the $\mu \to e\gamma\gamma$ and $\mu \to ee\bar{e}$ decays, see Eqs. (12), (14), (17), are essentially less then the existing experimental limits

$Br(\mu \to e\gamma\gamma)_{exp} < 7.2 \cdot 10^{-11}$ [17],

$Br(\mu \to ee\bar{e})_{exp} < 1.0 \cdot 10^{-12}$ [18],

they are not as small as the Standard Model predictions, and a hope for their observation in the future still remains.

In our opinion, the results obtained could be of interest for the discussions of the prospects of further searches for the $\mu \to e\gamma$, $\mu \to e\gamma\gamma$, and $\mu \to ee\bar{e}$ decays.

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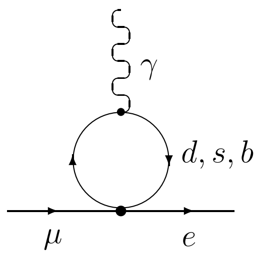
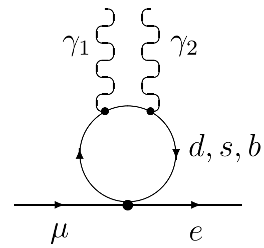
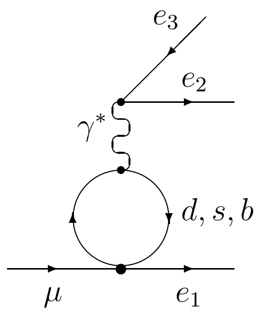


Fig. 1.



$+ \quad (\gamma_1 \leftrightarrow \gamma_2)$

Fig. 2.



$- \quad (e_1 \leftrightarrow e_2)$

Fig. 3.

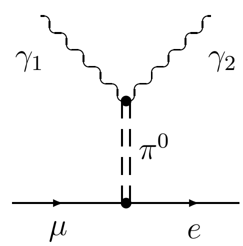


Fig. 4.